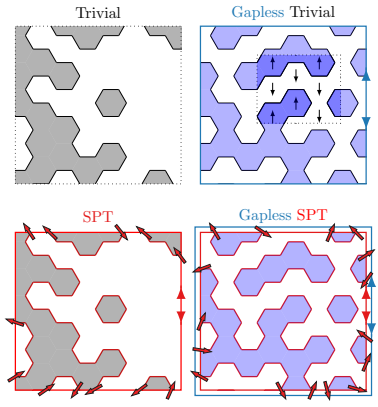
Gapless Symmetry Protected Topological Phases

APS March Meeting 2018

Thomas Scaffidi, DEP, Romain Vasseur.
Physical Review X. Nov 2017.

DEP, Thomas Scaffidi, Romain Vasseur.
arXiv:1711.09106



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Advisor

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UC Berkeley

SPT Phenomena

Symmetry Protected Topological phases are the simplest topological phases.

Mostly *classified* by Wen and many others.

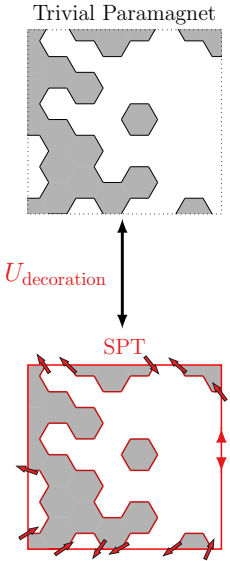
Famous Examples:

- topological insulators (fermionic)
- Haldane/AKLT chain (1D bosonic)

SPT Phenomena	
Protecting Sym. Group G	✓
Bulk Gap	✓
Gapless or S.B. Edge	✓
Short Range E.E.	✓
Gapless Entanglement Spectrum	✓
1D, 2D, 3D,...	✓
Bulk-Boundary Corr.	✓

SPTs do not need a bulk gap

Decorated Domain Wall Construction



Decoration is a local unitary operator U_{dec}.

It binds *charges* to *domain walls* and visa-versa.

Recursive Construction of SPTs:

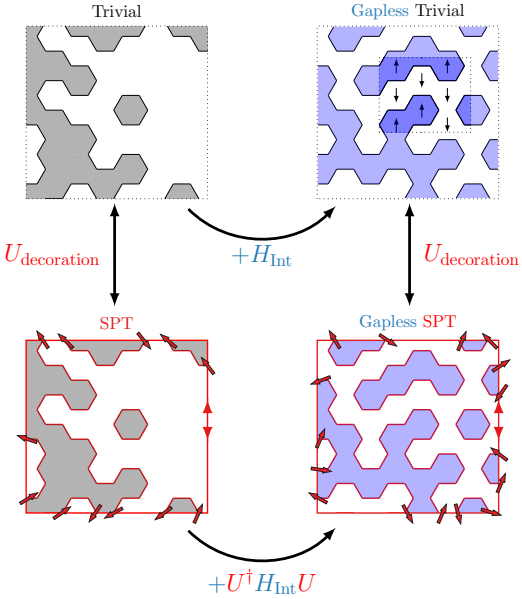
(I) Start with paramagnet H_{Trivial} in dimension d.

(II) Decorate domain walls with dim-$(d-1)$ SPT's:

$$H_{\text{SPT}} = U_{\text{dec}}^{\dagger} H_{\text{Trivial}} U_{\text{dec}}.$$

X. Chen, Y.M. Lu, A. Vishwanath. Nat. Comm., 2014.

Gapless SPT Order: Construction



(I) Start with H_{Trivial} and a twist U such that $H_{\text{SPT}} = U_{\text{dec}}^{\dagger} H_{\text{Trivial}} U_{\text{dec}}$ is an SPT.

T. Scaffidi, DEP, R. Vasseur. Physical Review X. Nov 2017.

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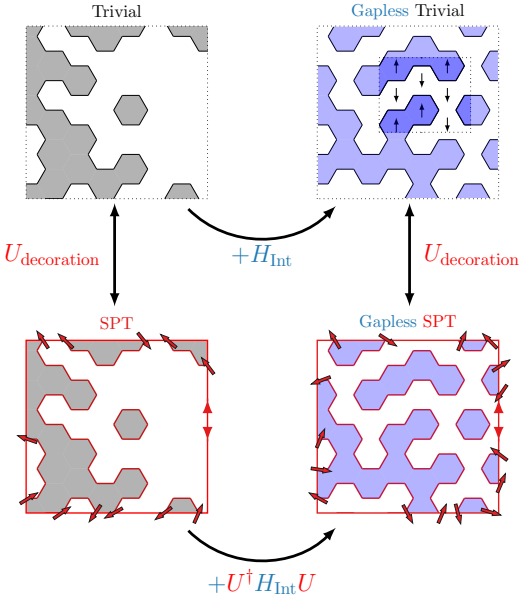

(I) Start with H_{Trivial} and a twist U such that $H_{\text{SPT}} = U_{\text{dec}}^{\dagger} H_{\text{Trivial}} U_{\text{dec}}$ is an SPT.

(II) Add a perturbation H_{Int} so that $H_{\text{Gapless}} = H_{\text{Trivial}} + H_{\text{Int}}$ is gapless.

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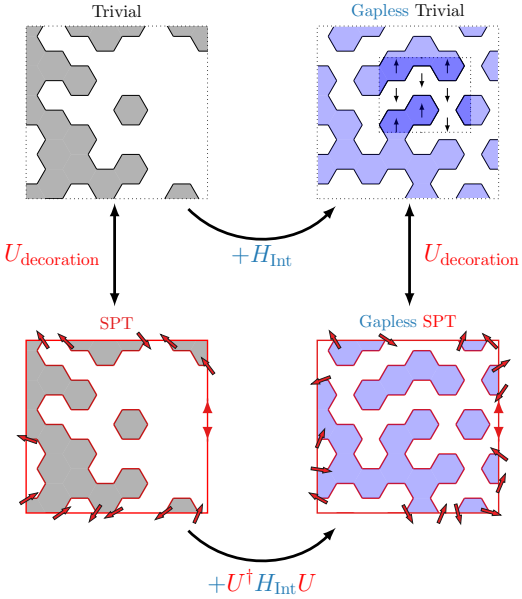

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(III) Twist the gapless model:

$$H_{\text{gSPT}} = U_{\text{dec}}^{\dagger} H_{\text{Gapless}} U_{\text{dec}}.$$

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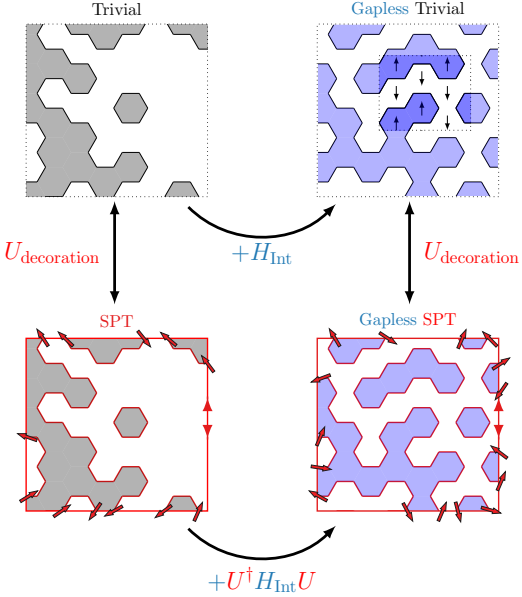

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$$H_{\text{gSPT}} = U_{\text{dec}}^{\dagger} H_{\text{Gapless}} U_{\text{dec}}.$$

If H_{Gapless} describes a gapless point/line/phase, then H_{gSPT} will be a gSPT point/line/phase.

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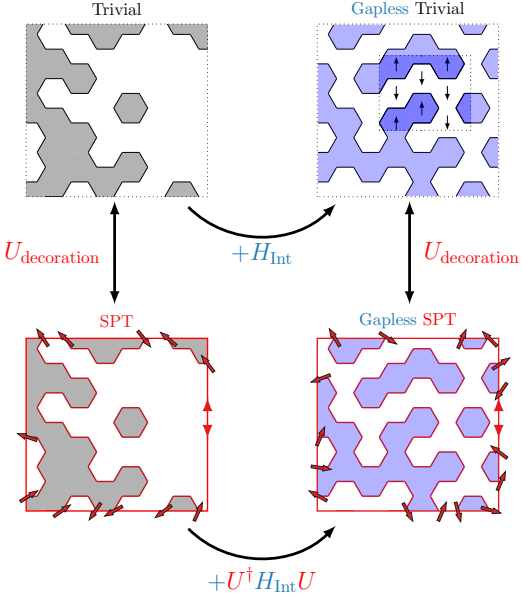


Phenomena	SPT	gSPT
Protecting Sym. G	✓	✓
Gapless Bulk	✗	✓
Gapless or S.B. Edge	✓	✓
Gapless Entanglement Spectrum	✓	✓
1D, 2D, 3D,...	✓	✓
Bulk-Boundary Corr.	✓	✓

Gapless SPT Zoology

1D Ising $\mathbb{Z}_2 \times \mathbb{Z}_2$ gSPT

1D Luttinger Liquid $\mathbb{Z}_2^2 \times U(1)$ gSPT

2D $\mathbb{Z}_2^3$ gSPT

T. Scaffidi, DEP, and R. Vasseur. Phys. Rev. X. Nov 2017.
DEP, T. Scaffidi, R. Vasseur. arXiv:1711.09106

Topological Superconductors

A. Keselman, E. Berg. Phys. Rev. B. June 2015.
A. Keselman, E. Berg, P. Azaria. arXiv:1802.02316

Unusual Surface Criticality

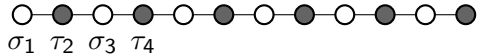
L. Zhang, F. Wang. Phys. Rev. Lett. Feb 2017.
C. Ding, L. Zhang, W. Guo. arXiv: 1801.10035.

SPT-SPT transitions

L. Tsui, H.C. Jiang, Y.M. Lu, D.H. Lee. Nuc. Phys. B. July 2015.

Many others!

Example: Decorated Luttinger Liquid



- $\mathbb{Z}_2 \times \mathbb{Z}_2 =$ flip σ's $\times$ flip τ's.
- Luttinger liquid model:

$$H_{\text{gapless}} = H^{(\tau)}_{\text{paramagnet}} + H^{(\sigma)}_{\text{XXZ}}(\Delta) + H_{\text{int}}(\alpha)$$

$$H^{(\tau)}_{\text{paramagnet}} = \sum_i -\tau^x_i$$

$$H^{(\sigma)}_{\text{XXZ}} = \sum_i \sigma^x_i \sigma^x_{i+1} + \sigma^y_i \sigma^y_{i+1} + \Delta \sigma^z_i \sigma^z_{i+1}.$$

- Decoration U_{dec}:

$$\sigma^x_i \to \tau^z_{i-1} \sigma^x_i \tau^z_{i+1},$$
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and similarly for τ.

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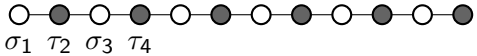

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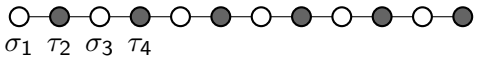

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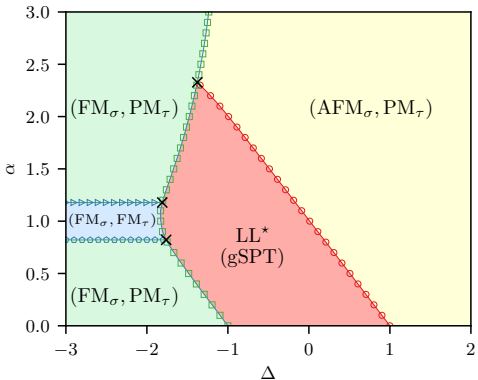

- Decoration U_{dec}:

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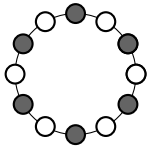
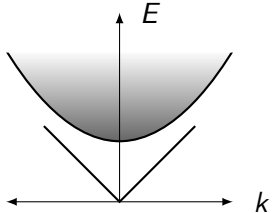
How gSPTs Work

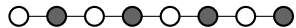
Periodic Boundary Conditions

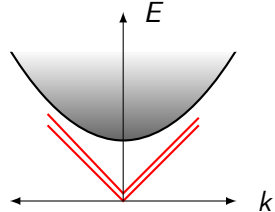




Unique ground state

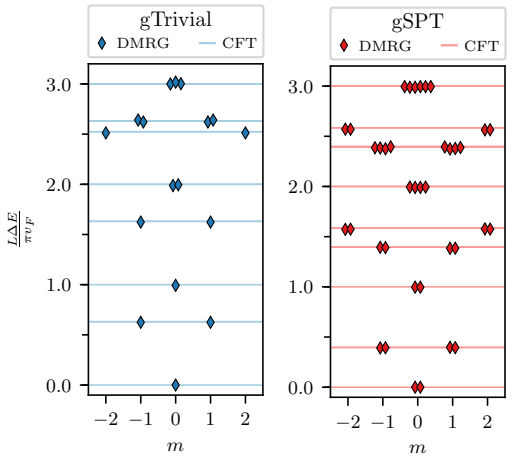
Open Boundary Conditions



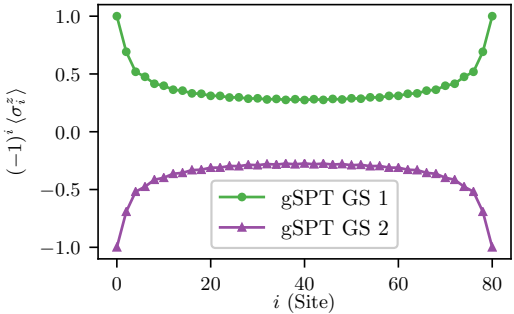


Multiple ground states
(topological edge modes!)

Example: Decorated Luttinger Liquid



$$\mathcal{L} = \frac{g}{4\pi}(\partial\varphi)^2 \qquad \mathcal{L} = \frac{1}{16\pi g}(\partial\varphi)^2$$



- Spotaneous symmetry-breaking at the *edge*.

$$|GS_\pm\rangle = |\uparrow_1, \Psi_{\text{bulk}}, \uparrow_L\rangle \pm |\downarrow_1, \Psi_{\text{bulk}}, \downarrow_L\rangle$$

- Described in boundary conformal field theory as a unusual *superposition* of conformal boundary conditions.

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Conclusions and Outlook

- The properties of SPTs can persist even without a bulk spectral gap.
- gSPTs found in a variety of contexts
- Gapless SPTs blend SPT physics with critical/gapless physics.
- This could be a way to systematically understand gapless topological matter.

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